MF03 AMPERE'S LAW & ELECTROMAGNETIC INDUCTION

SPH4U

CH 8 (KEY IDEAS)

- define and describe concepts related to magnetic fields
- compare and contrast the properties of electric, gravitational, and magnetic fields
- predict the forces on moving charges and on a current-carrying conductor in a uniform magnetic field
- perform and analyze experiments and activities on objects or charged particles moving in magnetic fields
- analyze and explain the magnetic fields around coaxial cables
- describe how advances in technology have changed scientific theories
- evaluate the impact of new technologies on society

EQUATIONS

- Ampere's Law

$$\Sigma B_{\parallel} \Delta l = \mu_0 I$$

- Straight Conductor (wire)

$$B = \mu_0 \left(\frac{I}{2\pi r} \right)$$

- Solenoid

$$B = \mu_0 \left(\frac{NI}{L} \right)$$

UNIFORM MAGNETIC FIELDS

- So far, we've been working with uniform magnetic fields, such as within the core of a solenoid or between the poles of a horseshoe magnet
- We can find their magnitude by finding the magnetic force
- Moving Charge

$$B = \frac{F_M}{qv\sin\theta}$$

- Conductor

$$B = \frac{F_M}{Il\sin\theta}$$

NON-UNIFORM MAGNETIC FIELDS

- When magnetic fields aren't uniform, we can see that $B \propto I$ and $B \propto \frac{1}{r}$, or

$$B = k\frac{I}{r}$$

  - B – magnetic field [T]
  - I – current [A]
  - r – distance from conductor [m]
  - k – proportionality constant

AMPÈRE'S LAW

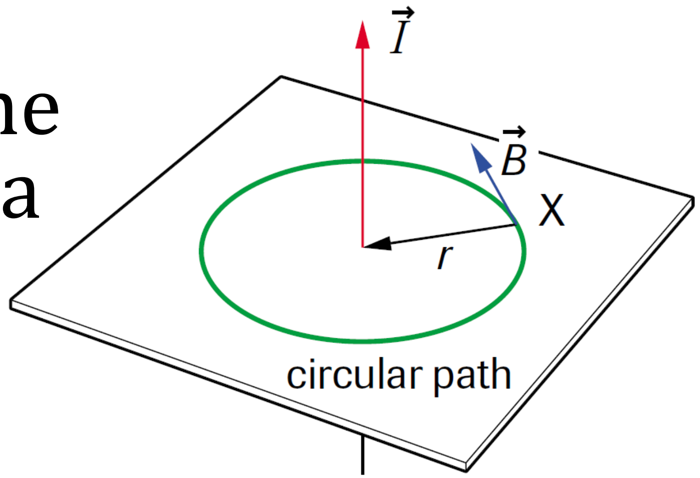
- André Marie Ampère developed the relationship between the current in any conductor and the strength of a magnetic field it produces
- **Ampère's Law:** Along any closed path through a magnetic field, the sum of the products of the scalar component of $\vec{B}$, parallel to the path segment with the length of the segment, is directly proportional to the net electric current passing through the area enclosed by the path.

$$\sum \boldsymbol{B}_{\parallel} \Delta \boldsymbol{l} = \boldsymbol{\mu_0 I}$$

  - $B_{\parallel}$ – component of B parallel to a segment of the path, Δl
  - Δl – a small segment into which the field is divided; tangent to a curve
  - I – the net current flow through the area enclosed by the path
  - μ_0 – constant; permeability of free space ($4\pi \times 10^{-7}$ T m/A)

AMPÈRE'S LAW: STRAIGHT CONDUCTOR (WIRE)

- Consider a long, straight conductor with current I
- The magnetic field B will take a circular path around the wire of radius r
- Δl is a small segment of the conductor, considered to be tangent to the circular path
- $B_{\parallel} = B =$ constant since the magnitude of the magnetic field around a wire is constant for a given distance, and $\vec{B}$ is parallel to Δl



AMPÈRE'S LAW
STRAIGHT CONDUCTOR (WIRE) – CONT.

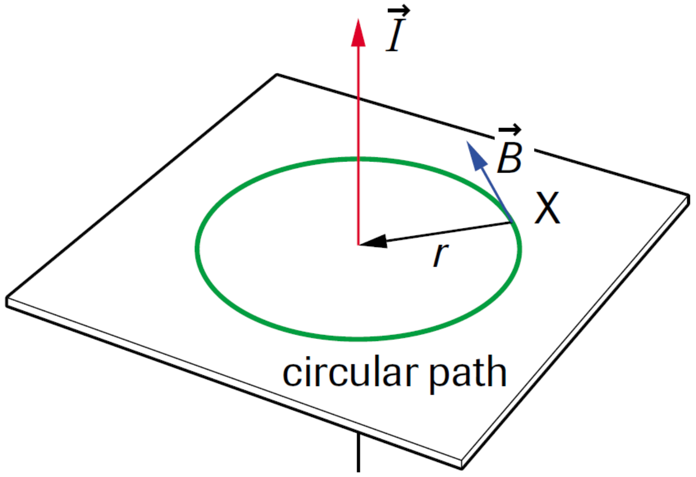
- Since $l = \sum \Delta l = 2\pi r$ for the circle,

$$\sum B_{\parallel} \Delta l = \sum B \Delta l = B \sum \Delta l = B(2\pi r) = \mu_0 I$$

- Ampère's Law for a straight conductor:

$$\boldsymbol{B} = \boldsymbol{\mu_0} \left(\frac{\boldsymbol{I}}{\boldsymbol{2\pi r}} \right)$$

  - B – magnetic field [T]
  - μ_0 – permeability of free space ($4\pi \times 10^{-7} \frac{\mathrm{Tm}}{\mathrm{A}}$)
  - I – current [A]
  - r – radius of curvature [m]



PROBLEM 1

What is the magnitude of the magnetic field 2.0 cm from a long, straight conductor with a current of 2.5 A?

PROBLEM 1 – SOLUTIONS

$r = 2.0 \text{ cm} = 0.020 \text{ m}$

$I = 2.5 \text{ A}$

$B = ?$

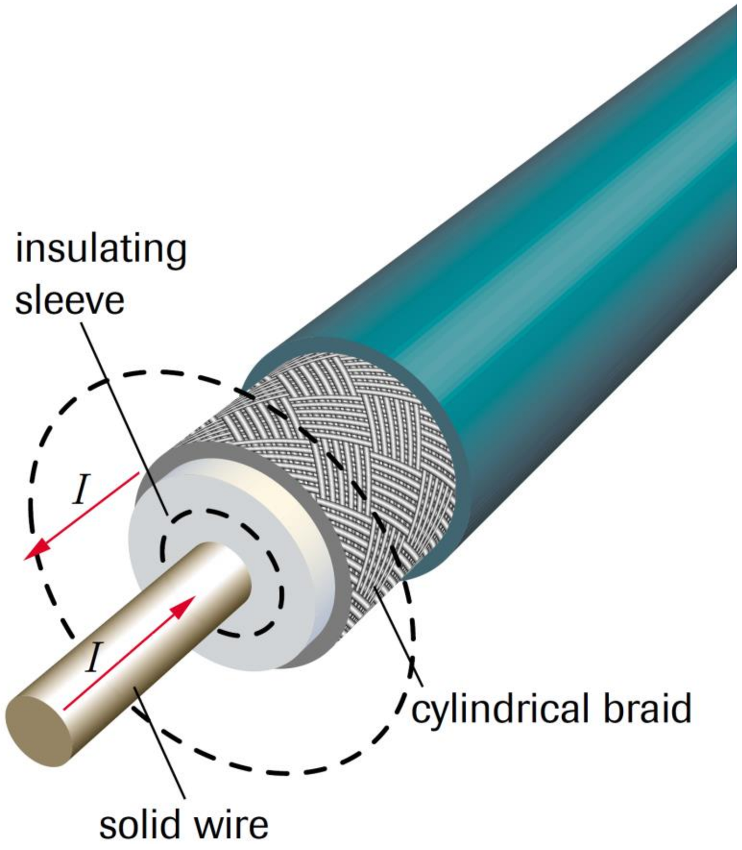
$$B = \mu_0 \left(\frac{I}{2\pi r} \right)$$

$$= \frac{(4\pi \times 10^{-7} \text{ T·m/A})(2.5 \text{ A})}{2\pi (0.020 \text{ m})}$$

$$B = 2.5 \times 10^{-5} \text{ T}$$

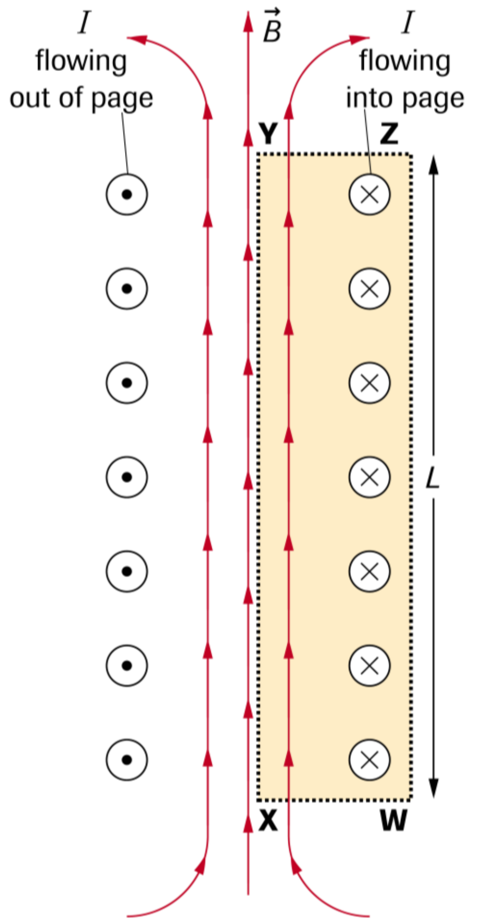
COAXIAL CABLES AND MAGNETIC FIELDS

- Consider the solid inner wire
  - Current *I* generates a magnetic field in the space between the inner wire and outer cylindrical braid
  - A second current *I* travels in the opposite direction along the cylindrical braid, generating a magnetic field in the opposite direction
- The magnetic fields cancel, resulting in a net magnetic field of zero outside the cable



AMPÈRE'S LAW: SOLENOID

- Consider the magnetic field of a solenoid
- Let's choose the rectangular path WXYZ; we can look at each side individually
- Magnetic fields for each segment are
  - WX: perpendicular to field; $B_{\parallel} = 0$
  - YZ: perpendicular to field; $B_{\parallel} = 0$
  - ZW: zero field; $B_{\parallel} = 0$
  - XY: $B_{\parallel} = B = \text{constant}$
- The segment $\text{XY} = L$, the length of the solenoid



AMPÈRE'S LAW: SOLENOID – CONT.

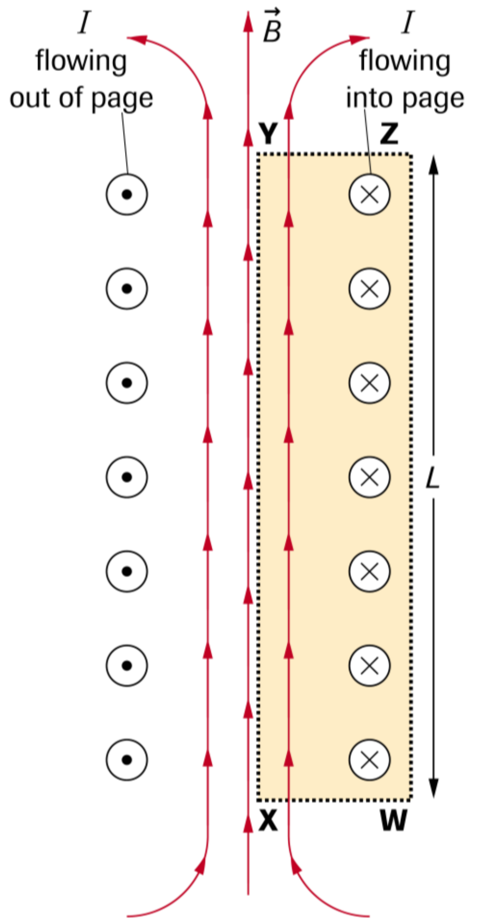
- In the area bound by path WXYZ, Ampère's Law becomes

$$\sum B_{\parallel}\Delta l = 0 + 0 + 0 + BL = \mu_0 NI$$

- Ampère's Law for a Solenoid:

$$\boldsymbol{B = \mu_0\left(\frac{NI}{L}\right)}$$

  - B – magnetic field [T]
  - μ_0 – permeability of free space $(4\pi \times 10^{-7}\,\frac{\text{Tm}}{\text{A}})$
  - N – number of coils
  - I – current in the wire [A]
  - L – length of the solenoid [m]



PROBLEM 2

What is the magnitude of the magnetic field in the core of a solenoid 5.0 cm long, with 300 turns and a current of 8.0 A?

PROBLEM 2 – SOLUTIONS

$L = 5.0\text{ cm} = 5.0 \times 10^{-2}\text{ m}$

$N = 300$

$I = 8.0\text{ A}$

$B = ?$

$$B = \mu_0\left(\frac{NI}{L}\right)$$

$$= \frac{(4\pi \times 10^{-7}\text{ T·m/A})(300)(8.0\text{ A})}{5.0 \times 10^{-2}\text{ m}}$$

$$B = 6.0 \times 10^{-2}\text{ T}$$

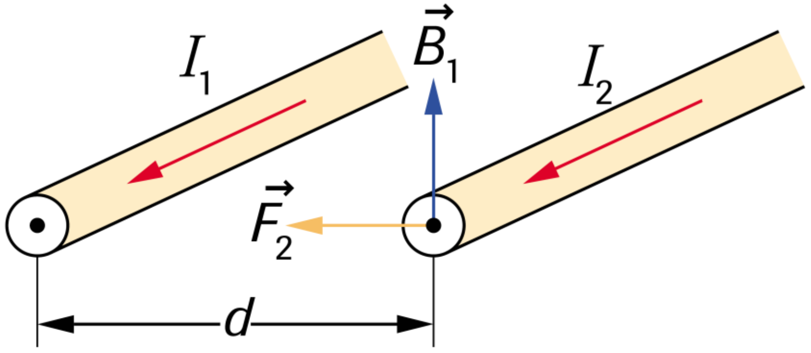
The magnitude of the magnetic field is 6.0×10^{-2} T.

THE AMPERE:
A UNIT OF ELECTRIC CURRENT

- **Current (I) [A = C/s]:** the amount of charge passing by a point in a given time
- Consider two long straight parallel conductors in a vacuum
- The magnetic field created by wire 1

$$B_1 = \mu_0 \left(\frac{I_1}{2\pi d} \right)$$

  - μ_0 – proportionality constant $(4\pi \times 10^{-7} \frac{\mathrm{Tm}}{\mathrm{A}})$
  - I_1 – current in wire 1 [A]
  - d – distance between the wires [m]



THE AMPERE: A UNIT OF ELECTRIC CURRENT – CONT.

- Recall: $F_M = IlB \sin\theta$
- The magnitude of the force acting on wire 2 is

$$F_2 = I_2 l B_1 \sin\theta$$

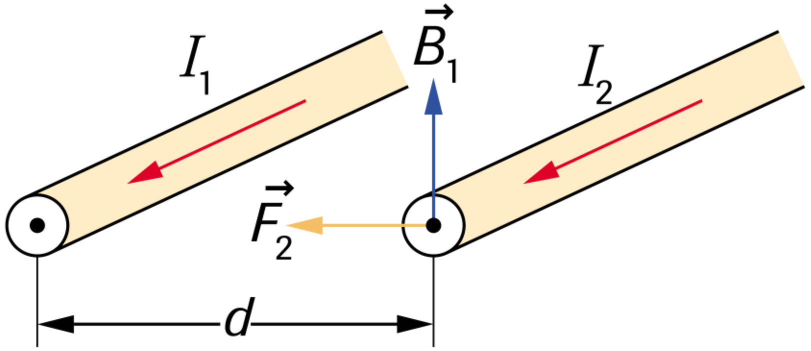
- Since the field is perpendicular, $\sin\theta = 1$, so

$$F_2 = I_2 l B_1$$

- Subbing in B_1, we get

$$F_2 = I_2 l \left(\mu_0 \left(\frac{I_1}{2\pi d} \right) \right)$$

$$F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$



THE AMPERE: A UNIT OF ELECTRIC CURRENT – CONT.

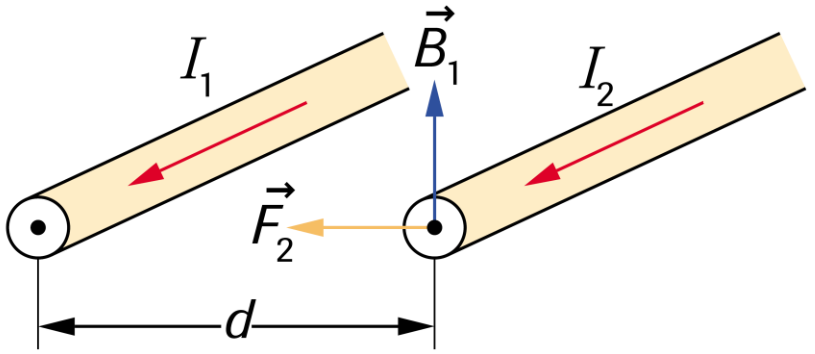
- The force per unit length is then

$$\frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

- To find the value of an ampere, we choose $d = 1$ m, $I_1 = I_2 = 1$ A, to get

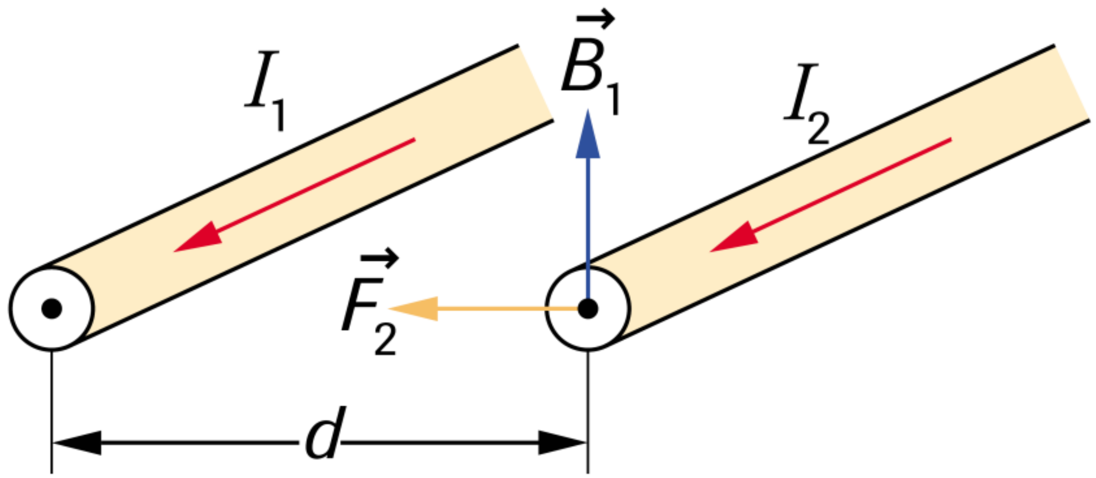
$$\frac{F}{l} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T}\frac{\mathrm{m}}{\mathrm{A}}\right)(1\ \mathrm{A})(1\ \mathrm{A})}{2\pi(1\ \mathrm{m})}$$

$$\frac{F}{l} = 2\times10^{-7}\ \mathrm{N/m}$$



THE AMPERE: A UNIT OF ELECTRIC CURRENT – CONT.

- **Ampere [A]:** SI unit of electric current; $\frac{F}{l} = 2 \times 10^{-7}$ N/m
- **Coulomb [C]:** SI unit of electric charge; 1 C = 1 A/s = N/m s



PROBLEM 3

What is the magnitude of the force between two parallel conductors 2.0 m long, with currents of 4.0 A and 10.0 A, 25 cm apart in a vacuum?

PROBLEM 3 – SOLUTIONS

$l = 2.0\ \text{m}$

$I_1 = 4.0\ \text{A}$

$I_2 = 10.0\ \text{A}$

$d = 25\ \text{cm} = 0.25\ \text{m}$

$F = ?$

$$\frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

or

$$F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T·m/A})(4.0\ \text{A})(10.0\ \text{A})(2.0\ \text{m})}{2\pi(0.25\ \text{m})}$$

$$F_2 = 6.4 \times 10^{-5}\ \text{N}$$

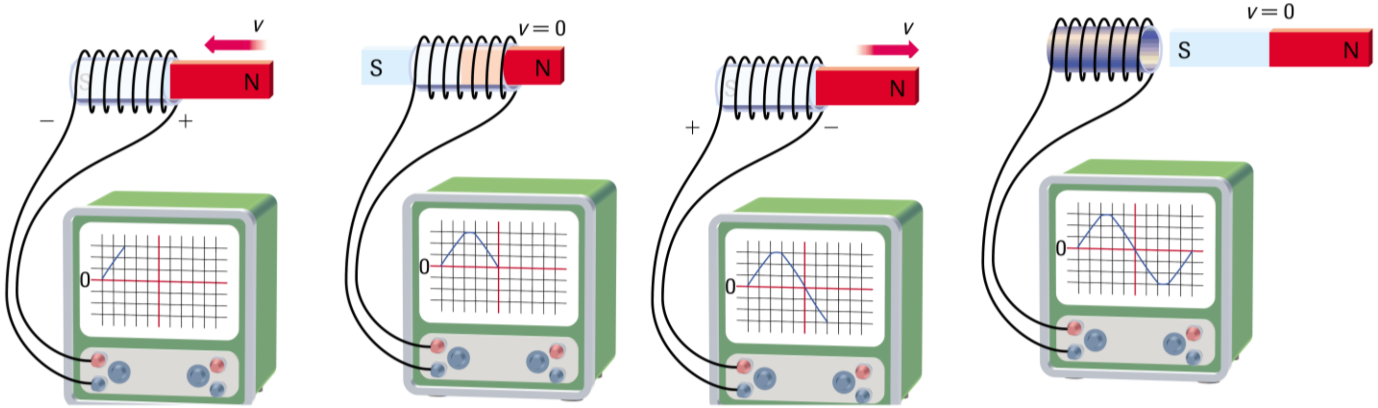
The magnitude of the force between the two parallel conductors is 6.4×10^{-5} N.

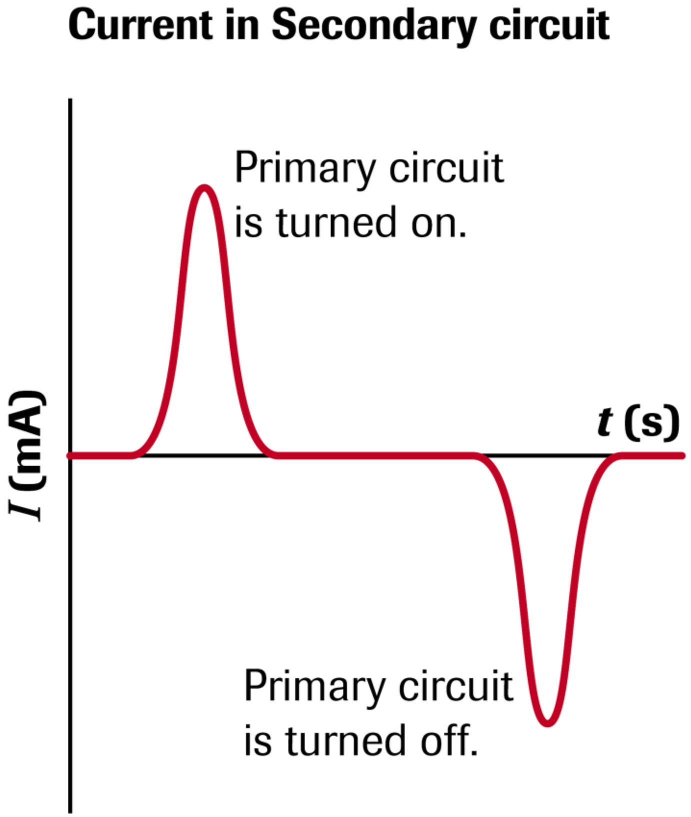
INDUCING ELECTRIC CURRENT

- We have just explored how a constant electric current can produced a constant magnetic field
- However, a constant magnetic field does not produce a constant current!
- Faraday explored this phenomenon and developed the Law of Electromagnetic Induction

LAW OF ELECTROMAGNETIC INDUCTION

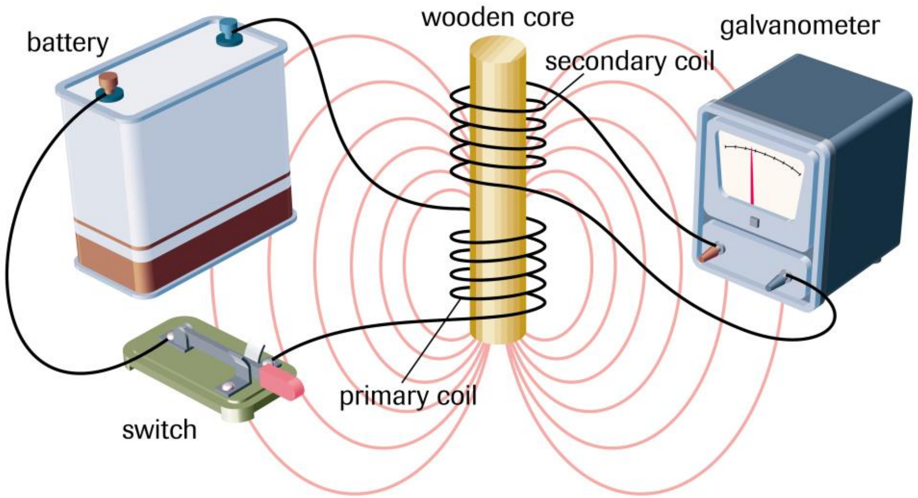
- **Law of Electromagnetic Induction:** An electric current is induced in a conductor whenever the magnetic field in the region of the conductor changes with time.

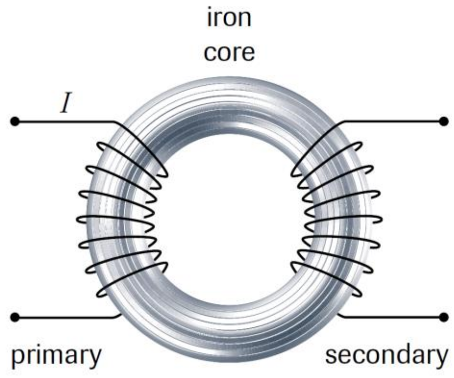




FARADAY'S EXPERIMENTS

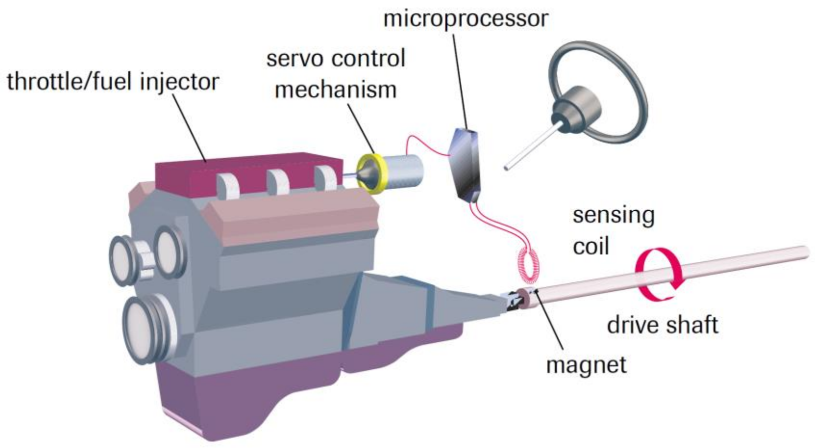
- First, Faraday used a wooden core to connect two solenoids
  - The charges do not pass between the circuits
- Then, he increase the magnitude of the magnetic field by using an iron core, in a "Faraday Ring"
- In both cases, the current is only induced in the secondary circuit when the primary circuit is switched on or off

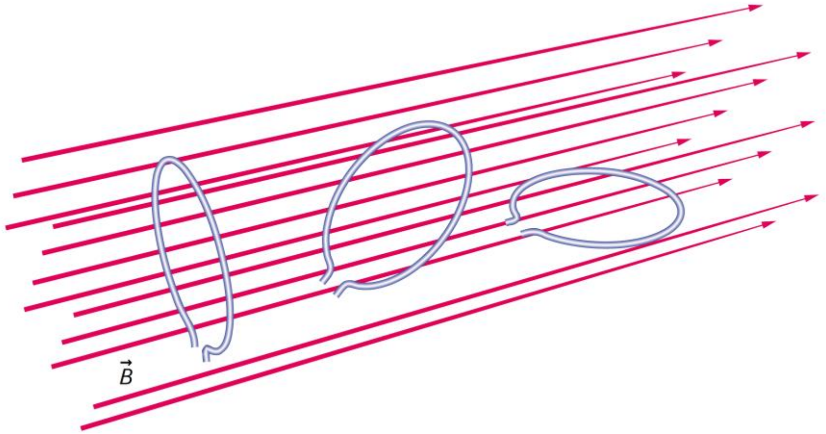




APPLICATION: CRUISE CONTROL

- Cruise Control in a vehicle uses the change in current of a rotating ring to determine the speed of the vehicle
  - It can send a signal to add more fuel when it detects the ring frequency is decreasing, less if increasing
- Automatic Breaking Systems and Traction Control use similar mechanisms





LENS' LAW

- There are two ways current can be induced in a solenoid
- The induced current will create its own magnetic field, as per our previous right-hand rule
- One direction induces the reverse pole, which would attract the magnet
  - This violates the conservation of energy
- The other direction creates the same pole, which repels the magnet

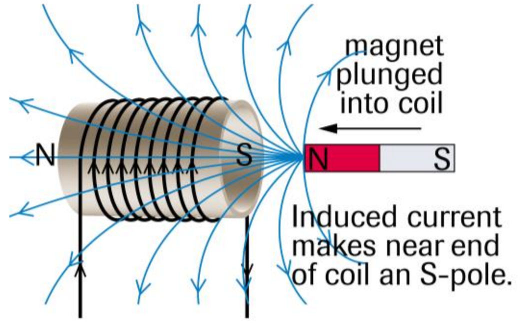


Figure 7
Violating the law of conservation of energy

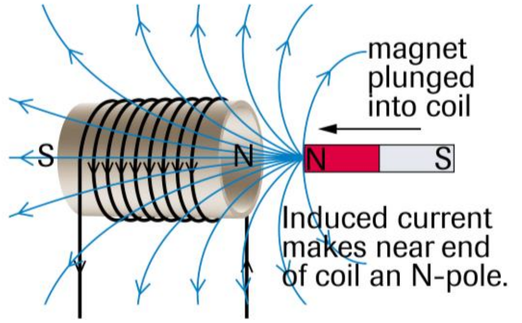
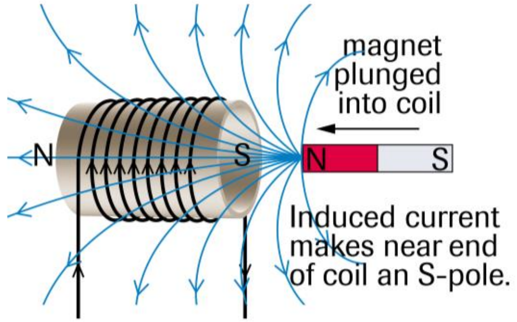


Figure 8
Obeying the law of conservation of energy

LENS' LAW

- **Lens' Law:** When a current is induced in a coil by a changing magnetic field, the electric current is in such a direction that its own magnetic field opposes the change that produced it.
- Loss of energy by the magnet by the induced magnetic field is energy gained by the current



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Violating the law of conservation of energy

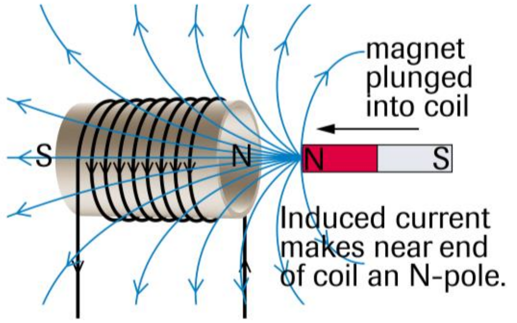

Figure 8
Obeying the law of conservation of energy

APPLYING LENS' LAW

- Maglev Trains:
  - Superconducting electromagnets on the bottom of the train create a force in the coils on the railway
  - This induces an upward force, levitating the train
- Induction Cooktops
  - Currents can be induced within a conducting plate (no coils necessary)
  - This can be seen in the <u>eddy currents</u> formed when using an induction cooktop and a metal pot
  - The alternating current used to power the cooktop operate the electromagnets, constantly changing the magnetic field and inducing a current in the pot (a conductor)
  - The resistance of the metal causes the pot to heat up, boiling the water

SUMMARY – AMPERE'S LAW

- Ampère's law states: $\sum B_{\parallel} \Delta l = \mu_0 I$.
- SI defines an ampere as the current in each of two long, straight, parallel conductors 1 m apart in a vacuum, when the magnetic force between them is 2×10^7 N per metre of length.
- SI defines the coulomb as the charge transported by a current of 1 A in a time of 1 s.

SUMMARY – ELECTROMAGNETIC INDUCTION

- The law of electromagnetic induction states that an electric current is induced in a conductor whenever the magnetic field in the region of the conductor changes.
- The greater the change in the magnetic field per unit time, the larger the induced current.
- Lenz's law states that when a changing magnetic field induces a current in a conductor, the electric current is in such a direction that its own magnetic field opposes the change that produced it.

PRACTICE

Readings

- Section 8.4 (pg 408)
- Section 8.5 (pg 415)

Questions

- pg 414 #1-6
- pg 419 #1-4